UK-06-01

Loop Corrected D-brane Stability Conditions

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Abstract

In type-II string theory compactifications on Calabi-Yau manifolds, topological string theory partition functions give a class of exact F-terms in the four dimensional effective action. We point out that in the background of constant self-dual field strength, these terms deform the central charges for D-branes wrapping Calabi-Yau manifold to include string loop corrections. We study the corresponding loop corrected D-brane stability conditions, which for B-type branes at the large volume limit implies loop corrected Hermitian-Yang-Mills equation, and for A-type branes imply loop corrected special Lagrangian submanifold condition.

1 Introduction

BPS states from D-branes wrapping curved space, in particular Calabi-Yau manifold, have been among the most important subjects of string theory research ever since the second string theory revolution. The effective D=4 $\mathcal{N}=1$ theories obtained from such brane configuration occupy a large sector of the string theory phenomenology and provide promising candidates for reality. The mathematical structures obtained from the brane picture is quite rich, for example the Kontsevich mirror symmetry conjecture. Also they provide a microscopic picture for BPS blackholes and many extraordinary results like blackhole entropy counting have been obtained.

The physics of D-branes wrapping Calabi-Yau has distinctive feature of the decoupling of the Kähler moduli and the complex structure moduli at generic points of the moduli space. In the effective D=4 $\mathcal{N}=2$ supergravity coupled with matters, one moduli underlies the vector multiplets and the other underlies the hypermultiplets, and the decoupling of these two moduli at generic points of the moduli space is the requirement of the $\mathcal{N}=2$ supersymmetry. From the super-Yang-Mills theory on the branes wrapping the internal space and expanding the four-dimensional spacetime, one moduli underlies the superpotential while the other underlies the supersymmetric D-term, depending on either IIA or IIB branes. This is the basis of the category description of the all type II D-branes, as pioneered by M. Douglas and collaborators [1, 2], that the BPS branes are objects in a category determined by complex structure for B-type branes and Kähler structure for A-type branes. Then the grading is defined at every point in the other half of the moduli space, i.e. the Kähler moduli for B-type branes and complex structure moduli for A-type branes, and used to define stable objects.

It has been argued that, since the dilaton field sits in one of the hypermultiplets, the part of the Lagrangian involving vector multiplets would not receive string loop correction, as the consequence of the decoupling conjecture. Thus the BPS mass formula is exact at the tree level string theory[4]. This corresponds to use only the genus-zero partition function of the twisted topological string theory as the prepotential for vector multiplets in the effective supergravity theory.

However, more exact F-terms of the form $F_g W^{2g}$ are present in the effective theory, all depend on powers of the $\mathcal{N}=2$ graviphoton superfield W, and explicitly depend on the high genus partition function of the twisted topology string theory F_g [8, 9]. In the effective action, they produce high derivative interactions, and blackhole solutions and entropy countings have been studied extensively to produce string loop corrected results [6, 7].

We will notice that in the presence of constant graviphoton field strength λ^2, these terms produce a series expansion in λ^2 which corrects the prepotential for vector supermultiplets.

Combined with the corrected central charge formula, we will see that they produce string loop corrected stability conditions for BPS D-branes wrapping Calabi-Yau manifold. In particular, this implies string loop corrected Hermitian-Yang-Mills equation in the large volume limit for B-type branes, and loop corrected special Lagrangian submanifold condition for A-type branes. We will compare the known loop corrections to HYM equation in heterotic string theory to support this claim.

It may seem strange that a string loop expansion should be controlled by the graviphoton field strength. Similar phenomena has occurred in the computation of the quantum corrected glueball superpotential of the super-Yang-Mills theory, and has been studied in [5] and called C-deformation. The stability condition we propose in this paper can be viewed as the effect on D-term in the super-Yang-Mills theories obtained from the D-branes wrapping Calabi-Yau manifold from such C-deformation, as in contrast to the effect on superpotential in [5]. The similarity between these two is quire interesting.

The plan of the paper is as follows. After the introduction, we examine the $D = 4, \mathcal{N} = 2$ effective supergravity action with high derivative terms, and show that constant graviphoton field strength deforms the moduli space geometry. Then after a conformal field theory argument that graviphoton field strength can be non-zero in string vacuum, we study a toy model of high derivative pure supergravity, in which effective potential for graviphoton field strength is present and constraints the value of graviphoton field strength in the vacuum depending on the gravity background. Then we study the central charge formula which includes the high loop corrections. In the next section, we study the consequences of the loop corrected D-brane stability conditions due to central charge formula. For B-type branes in the large volume limit, loop correction produces corrected Hermitian-Yang-Mills equation, which matches the known loop corrected heterotic string result after type I-heterotic duality. For A-type branes, the BPS brane is found to wrap loop corrected special Lagrangian submanifolds. The last section contains conclusions and discussions.

2 Loop corrected central charge

We first review the effective $D = 4, \mathcal{N} = 2$ supergravity action coupled with vector multiplets, together with the high derivative terms. We will see the deformation of the moduli geometry in the presence of the constant graviphoton field strength. We then look at the determinination of the possible value of the background constant graviphoton field strength in vacuum. It is this value controls the genus expansion of the central charge, and the symplectic sections. The expectation is that it can take arbitrary value, based on the conformal field theory argument. However, in the effective action, the supersymmetry requires the

presence of potential for chiral background, which fix it to certain value depending on the gravity background and dynamics. Finally we present the deformed central charge formula.

2.1 High derivative $D = 4, \mathcal{N} = 2$ effective theory and deformed moduli geometry

We consider the $D = 4$ effective theory of $\mathcal{N} = 2$ conformal supergravity obtained from type-II string theory compactification on Calabi-Yau manifold. Upon gauge fixing, it produces the Poincare supergravity. The fields in the theory include Weyl multiplet $\mathcal{W}^{ij}_{\alpha\beta}$ whose lowest component is the self-dual graviphoton field strength T^{ij}_{ab}, coupled with (n+1) abelian vector multiplets $X^I (i = 0, \cdots, n)$ whose lowest components are scalars, which we again designated by X^I. The hypermultiplets decouple from the vector multiplets, thus will not concern us in the following discussion, except the fact that the dilaton field sits in a hypermultiplet.

The effective action is constructed out of prepotential

$$\int d^4x d^4\theta \sum_g F_g(X^I)\mathcal{W}^{2g}, \tag{1}$$

where the integral is over half of the $\mathcal{N} = 2$ superspace coordinates, and $F_g(X^I)$ is a holomorphic function of the vector multiplets X^I's. In the string theory compactification on Calabi-Yau manifolds, $F_g(X^I)$ is equal to genus-g partition function of the twisted topological string theory on Calabi-Yau manifold [8, 9].

The integral over half of the superspace coordinates will produce various terms in the bosonic Lagrangian. We can absorb the θ coordinates by the Wely density $\mathcal{W}^2$, which gives coupling terms of vector multiplets scalars with gravity, or we can absorb the θ coordinates using vector multiples in $F_g(X^I)$, which only the scalar part of the Weyl density $\mathcal{W}^2$ appears. In the latter case, we obtain terms in the action as a series expansion in the graviphoton field strength λ^2

$$\int d^4x d^4\theta \sum_g F_g(X^I)\lambda^2, \qquad \lambda^2 \equiv \hat{A} = (T^{ij}_{\mu\nu}\epsilon_{ij})^2. \tag{2}$$

Notice that the action given by the $g = 0$ term F_0 is the usual prepotential for $\mathcal{N} = 2$ field theory of vector multiplets, and it can be also viewed as the result of setting $\lambda^2 = 0$ in the more general action. Now it is obvious that in presence of the constant graviphoton field strength, the prepotential is deformed to include terms corresponding to high genus topological string amplitudes, where the graviphoton field strength acts as deformation parameter and controls the genus expansion.

The problem of constant graviphoton field strength in the vacuum will be considered in the next section. Now we look at the implications to the geometry of the moduli space.

The geometry of the target space $\mathcal{M}$ parameterized by the scalars of the vector multiplets is a deformation of the usual $\mathcal{N} = 2$ special geometry. The full prepotential is again a homogeneous degree two holomorphic function,

$$F(cX^I, c^2\lambda^2) = c^2 F(X^I, \lambda^2), \tag{3}$$

where X^I and λ are degree-one sections of a line bundle over the moduli space. $2(n+1)$ degree-one sections over the moduli space can be obtained from the prepotential and again form a section of a rank $2(n+1)$ symplectic vector bundle over the moduli space $\mathcal{M}$,

$$\begin{pmatrix} X^I \\ F_I(X^I, \lambda^2) \end{pmatrix}. \tag{4}$$

Its symplectic inner product gives the deformed Kähler potential over the moduli space, from which the deformed metric can be obtained by taking appropriate derivatives

$$e^{-\mathcal{K}} = i[X^I \bar{F}_I(X^I, \lambda) - \bar{X}^I F_I(X^I, \lambda)], \tag{5}$$

$$g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} \mathcal{K}. \tag{6}$$

The corresponding Kähler form on the moduli space is the deformation in $H^2(\mathcal{M}, Z)$ of the one at $\lambda = 0$.

Let us look at the deformation in terms of the moduli space of the Calabi-Yau manifold. To be specific, we consider the moduli space of the complex structures of the Calabi-Yau manifold M, which is parameterized by the vector multiplets in the type IIA compactifications. The moduli space of the Kahler structures can be similarly obtained from the mirror symmetry.

At $\lambda = 0$, the symplectic section is obtained as follows. Take a symplectic basis of the homological three-cycles $\{A_\alpha, B_\beta\}$, $\alpha, \beta = 0, , 1, \cdots, n$ such that

$$A_\alpha \cap B_\beta = \delta_{\alpha\beta}, \qquad A_\alpha \cap A_\beta = 0, \qquad B_\alpha \cap B_\beta = 0, \tag{7}$$

where we assume $H^3(M, Z)$ has real dimension $2(n+1)$. Take holomorphic 3-form Ω on Calabi-Yau manifold, which is uniquely defined up to a scale, the pairing between the homology and the cohomology is given by the periods

$$X^I = \int_{A_I} \Omega, \qquad F_I = \int_{B_I} \Omega, \tag{8}$$

which satisfy Picard-Fuchs equation in toric Calabi-Yau manifolds.

One can define a deformed 3-form $\Omega(\lambda)$ according to the following property

$$X^I = \int_{A_I} \Omega(\lambda), \qquad F_I(X^I, \lambda^2) = \int_{B_I} \Omega(\lambda). \tag{9}$$

In general this prescription adds $H^{2,1}(M,C)$ piece to the holomorphic three-form. We will argue that this three-form should be used to calibrate the loop corrected special Lagrangian submanifolds.

Notice that here we assume the constant graviphoton field strength without specifying its value. This works as long as we do not consider the back reaction to the gravity. Certainly as will be evident in the toy model of pure supergravity in the following section, its value should be determined dynamically in specific gravity solutions.

2.2 Non-zero constant graviphoton field strength

To study the graviphoto photon field strength in vacuum, we first look at the conformal field theory argument, basically repeat the work in [5].

Utilizing the covariant quantization of string theory, the four-dimensional part of the worldsheet Lagrangian is

$$\mathcal{L} = \frac{1}{2}\partial X^\mu \bar{\partial} X_\mu + p_\alpha \bar{\partial}\theta^\alpha + p_{\dot{\alpha}} \bar{\partial}\theta^{\dot{\alpha}} + \bar{p}_\alpha \partial\bar{\theta}^\alpha + \bar{p}_{\dot{\alpha}} \partial\bar{\theta}^{\dot{\alpha}}, \tag{10}$$

where p's are worldsheet (1,0) forms, $\bar{p}$'s are (0,1) forms, and θ and $\bar{\theta}$'s are 0-forms. Turning on non-zero self-dual graviphoton field strength $T_{\alpha\beta}$ adds to the worldsheet the following term

$$\int F^{\alpha\beta} p_\alpha \bar{p}_\beta. \tag{11}$$

The worldsheet theory is still conformal invariant, so it still satisfies the string equation of motion.

From this argument, any value of the graviphoton field strength would be a vacuum solution. However, effective potential for graviphoton field strength could be generated from string theory higher loop or non-perturbatively. Actually it will be present in the effective theory, as we now turn to for a toy model of $D=4, \mathcal{N}=2$ pure supergravity action.

Toy model: constant graviphoton field strength in pure supergravity Now we turn to a toy model of pure $D=4$ $\mathcal{N}=2$ supergravity with high derivative corrections. One needs Weyl supermultiplet, one vector supermultiplet and one hypermultiplet/nonlinear multiplet/tensor multiplet (acts as compensator of the $SU(2)$ gauge group in superconformal action) to match the degree of freedom of the Poincare supergravity supermultiplet. The simplest action coupling to high derivative terms is to assume

$$F(X^0, \hat{A}) = \frac{-i}{4}(X^0)^2 + c\hat{A}. \tag{12}$$

Then the D-gauge fixing requires

$$e^{-K} = |X^0|^2 = 1, \tag{13}$$

while the A-gauge fixing $X^0 = \bar{X}^0$ requires

$$X^0 = \pm 1. \tag{14}$$

We will choose $X^0 = 1$ in the following. So indeed the scalar field in the vector multiplet is eliminated thorough the gauge fixing.

The bosonic part of the Lagrangian for this toy model, assuming the trivial behavior of the fields such as $SU(2)$ gauge field, the hypermultiplet field(decoupled from the vector multiplets), and the $U(1)$ gauge field, after gauge fixing to obtain Poincare supergravity, is

$$\begin{aligned} 8\pi e^{-1}\mathcal{L}_{\text{boson}} = & -\frac{1}{2}R + (32ic\mathcal{R}^{-\ \ cd}_{ab}\mathcal{R}^{-\ \ ab}_{cd} + h.c.) \\ & + 8i(c-\bar{c})(\mathcal{D}_a T^{-ab}\mathcal{D}^c T^+_{cb} + \frac{1}{2}R_a{}^c T^{-ab}T^+_{cb} + \frac{1}{128}\hat{A}\bar{\hat{A}}) \\ & + \frac{1}{8}(F^{-0} - \frac{1}{4}T^-)\cdot(F^{-0} - \frac{1}{4}T^-) - \frac{1}{16}(F^{-0} - \frac{1}{4}T^-)\cdot T^- \\ & - \frac{i}{32}[\frac{-i}{4}(X^0)^2 + c\hat{A}]\bar{\hat{A}} + h.c. \end{aligned} \tag{15}$$

Integrating out the vector field F^{-0} identifies T^- with the graviphoton field strength

$$F^{-0} = \frac{1}{2}T^{-0}, \tag{16}$$

and also produces a potential linear potential term for A. The Lagrangian reduces to

$$\begin{aligned} 8\pi e^{-1}\mathcal{L}_{\text{boson}} = & -\frac{1}{2}R + (32ic\mathcal{R}^{-\ \ cd}_{ab}\mathcal{R}^{-\ \ ab}_{cd} + h.c.) + 8i(c-\bar{c})\mathcal{D}_a T^{-ab}\mathcal{D}^c T^+_{cb} \\ & - \frac{1}{64}(A + \hat{A}) + 8i(c-\bar{c})(\frac{1}{2}R_a{}^c T^{-ab}T^+_{cb} + \frac{1}{128}\hat{A}\bar{\hat{A}}) \end{aligned} \tag{17}$$

Ignore the back reaction of the graviphoton field strength to the gravity, the vacuum could be any conformal background including the flat space. Now since there is a potential for the constant graviphoton field strength, exactly of the form of ϕ^4 potential for the scalar field, while the scalar is the chiral scalar field constructed from the graviphoton field strength $\hat{A}$. Notice that the Ricci-curvature appears as part of the mass for graviphoton field strength, so fixing graviphoton field strength is not decoupled from gravity. Even in the flat space background, the field graviphoton field strength is nonzero, and fixed at

$$\hat{A} = \frac{1}{4i(c-\bar{c})}. \tag{18}$$

In the dS or AdS space, the constant Ricci tensor acts as mass term for graviphoton field strength, which fixes its value with dependence on c and the curvature of the spacetime. There is a critical value of the curvature of the dS space above which the vacuum value of the graviphoton field strength is zero.

2.3 Deformed Central charge

In Poincare supergravity without high derivative corrections, the central charge for a blackhole carrying a set of electromagnetic charges (q_I, p^I) is defined as the integration of the spherical part of the graviphoton field strength $T^{-\theta\phi}$ at spatial infinity,

$$Z = \frac{1}{4\pi} \int_{S^2_\infty} dS T^-_{\theta\phi}. \tag{19}$$

In this case, T^-_{ab} is algebraically related to the symplectic invariant $U(1)$ projection of the $N+1$ vector fields

$$T^-_{ab} = F_I F^-_{ab} - X^I G^-_{ab}. \tag{20}$$

Thus there is no ambiguity to identify either as the the graviphoton field strength. The central charge, after solving the equation of motion for the vector fields and calculating the surface integral at spatial infinity, turns out to be

$$Z = e^{K/2}(p^I F_I - q_I X^I). \tag{21}$$

When high derivative terms are introduced through couplings to the background supergravity fields, there is yet no strict derivation of the central charge from the supergravity action. A natural candidate is simply

$$Z = e^{K(X^I,\lambda^2)/2}[p^I F_I(X^I, \lambda^2) - q_I X^I]. \tag{22}$$

Indeed this is the only symplectic invariant $U(1)$ projected expression, which reduces to the Poincare supergravity case when $\lambda^2 = 0$. A strict derivation should proceed from analyzing the supergravity action and obtained the supersymmetry algebra at spatial infinity. This is quite difficult, since in high derivative gravity the relation between T^-_{ab} and the $U(1)$ projection of all the vector fields is not algebraic anymore. These two feidls are related by complicated nonlinear equations which involve kinetic terms for T^{-ab} and back reaction to the metric is also present. One possible point of view of to regard the $U(1)$ projection $F_I F^-_{ab} - X^I G^-_{ab}$ as the physical graviphoton field strength, while T^-_{ab} as simply an auxiliary field whose value can only be determined dynamically.

A large class of blackhole solutions preserving $D = 4\mathcal{N} = 1$ symmetry are known, but asymptotically flat solutions seem to impose certain restriction on the moduli parameters X^I's at spatial infinity, and non-asymptotically solutions are abundant. They probably should still be called BPS blackholes as they are indeed solutions to the equation of motion and preserve half of the bulk supersymmetry. It is also suggested in [13] the non-asymptotically flatness is due to the fact that α' corrections can not be ignored at spatial

infinity and thus λ^2 corrections should show up at spatial infinity. Note that ADM mass for asymptotically AdS solutions is known, from which the central charge can be obtained from the BPS condition $|Z| = M$. This is important since that the class of BPS blackhole solutions with asymptotically flatness seems to require $\lambda^2 = 0$ at spatial infinity, while the $\mathcal{N} = 2$ solution with $AdS_2 \times S^2$ solutions when the moduli parameters sit at attractor point has $\lambda^2 \neq 0$ at spatial infinity. So asymptotic non-flat (maybe AdS or even dS) solutions should also be included, where λ^2 dependent terms in (eqn. of central charge) is expected to show up in the central charge formula. We notice that at least the solutions with full $D = 4$ $\mathcal{N} = 2$ symmetries exhibit such property.

A microscopic definition from the moduli space parameters also points to the correctness of above central charge formula. As we have seen in the previous discussion, the moduli space geometry in the presence of constant graviphoton field strength, for A-type branes the loop corrections are summarized by a λ^2 dependent three-form $\Omega(\lambda)$ from which symplectic sections are obtained . Thus the central charge formula above is the obvious one.

Another confirmation of this expression as α' and g_s loop corrected central charge in the presence of constant graviphoton field strength comes from its implications to D-brane stability condition, which could be verified with statements from other sources. This is what we will turn to in the next section.

3 Deformed Π-stability condition and SLAG

3.1 Loop corrected Π-stability condition and HYM equation

We first consider stability conditions for B-type BPS D-branes wrapping even dimensional cycles of Calabi-Yau manifold. Because of the decoupling of the vector multiplets and hypermultiplets, the $\mathcal{N} = 1$ supersymmetric configurations can be described in two steps. In the first step, BPS B-type branes are identified as objects in a category depends only on the complex structure of the Calabi-Yau manifold. In the large volume limit, it is the derived category of coherent sheaves on Calabi-Yau manifold. In the next step, each brane configuration determines a central charge at each point in the Kähler moduli space via its topological charges. Its phase defines a grading for each object in the category. Then Π-stability criterion for a BPS brane configuration is defined by comparing the phase of an object with that of all its sub-objects, which in the large volume limit becomes the μ stability condition of the holomorphic vector bundles.

This decoupling property is still preserved when α' and string loop corrections are included, as dictated by the $\mathcal{N} = 2$ $D = 4$ supersymmetry. Even in the presence of the background graviphoton field strength, the vector multiplets and the hypermultiplets are

decoupled in the effective action, as long as the vector fields are all abelian. Therefore we will single out the second step in determining the stability of the branes configuration, using the corrected central charge.

This in turn defines a loop corrected grading for type-B D-branes as an object E in the derived category of coherent sheaves[1]

$$\phi(E) = -\frac{1}{\pi}\mathrm{arg}Z(E) \tag{23}$$

Type-B branes wrap $2k$ cycles. In the large volume limit, the configuration can be described as a holomorphic vector bundle E, whose Chern classes give the topological charges

$$Q_6 = \mathrm{rank}(E) = r; \qquad Q_4 = \int_\Sigma c_1(E); \qquad Q_2 = \int_S ch_2(E); \qquad Q_0 = \int_S ch_3(E), \tag{24}$$

which can be read from the following formula

$$ch(E) = r + c_1 + \frac{1}{2}(c_1^2 - 2c_2) + \frac{1}{6}(c_1^3 - 3c_1c_2 + c_3) + \cdots . \tag{25}$$

They are associated with electric and magnetic charges separately,

$$p^I = (Q_6, Q_4), \qquad q_I = (Q_2, Q_0). \tag{26}$$

As we have seen in the previous section, the periods paired with various topological charges receive corrections as λ^2-dependent terms. To compare with known results, we look at the large volume limit where world sheet instanton contributions are small and can be ignored. The full prepotential assumes the form

$$F(t^i, \lambda) = \frac{1}{6}c_{ijk}t^it^jt^k - \frac{\lambda^2}{24}c_{ai}t^i, \tag{27}$$

where t^i parameterizes the complexified Kähler form

$$B + iJ = \sum_i t^i\omega_i, \tag{28}$$

the ω_i's being a basis of Kähler two forms. The coefficients c_{ijk} are triple intersection forms,

$$c_{ijk} = \int_M \omega_i \wedge \omega_j \wedge \omega_k. \tag{29}$$

[1]The definition here differs from that in [1] by a sign. It results from different conventions of the signs in the periods. In the large volume limit, it produces the same sign of the second term in the grading as in [1]. The differences in the grading is in the sign of the constant which is trivial, and the third term that we explicitly write out. This definition produces the right correction as the grading from the improved charge expression.

The corrected prepotential produces the symplectic basis at large volume limit ($J >> B$) as

$$\begin{aligned}
\Pi_0 &= 1; \\
\Pi_2^i &= t^i; \\
\Pi_4^i &= F_i(\lambda) = \frac{1}{2} c_{ijk} t^j t^k - \frac{\lambda^2}{24} c_{2i}, \\
\Pi_6 &= F(\lambda) = \frac{1}{6} c_{ijk} t^i t^j t^k - \frac{\lambda^2}{24} c_{2i} t^i.
\end{aligned} \tag{30}$$

This is based on the assumption that the the coordinate designation should be reduced to the tree level result when set $\lambda = 0$. This should serve as a definition of the generalized special geometry coordinates. Note the central charge is

$$Z(E) = e^{K/2}(p^I F_I(X^I, \lambda) - q_I X^I), \tag{31}$$

where the factor $e^{K/2}$ is real and has not effect on the phase. The expansion of the grading in the large volume limit is

$$-\frac{1}{\pi} \mathrm{Im} \log Z(E) = -\frac{3}{2} + \frac{6}{\pi} \frac{\frac{1}{2}\int c_1(E) J^2 + \frac{\lambda^2}{24}\int c_1(E) \wedge c_2(T) + \frac{1}{6}(c_1^3 - 3c_1 c_2 + c_3)}{rV} + \cdots . \tag{32}$$

The α' and loop corrected slope μ is

$$\mu(E) = \frac{1}{rV}[\int c_1(E) J^2 + \frac{\lambda^2}{12} \int c_1(E) \wedge c_2(T) + \frac{1}{3} \int (c_1^3 - 3c_1 c_2 + c_3)]. \tag{33}$$

Now the α' and string loop corrected Π-stability condition can be stated as follows: an object E is Π-stable at a point in Kähler moduli space if and only if for any subobject $E' \subset E$ satsifies

$$\phi(E') < \phi(E). \tag{34}$$

From this result, one notices the followings:

1. It has been known for some time that the charge for D-branes in curved space is

   $$Q(E) = ch(E)\sqrt{\hat{A}} \tag{35}$$

   where $\hat{A}$ is the A-roof-genus of a spin manifold,

   $$\sqrt{\hat{A}} = 1 + \frac{1}{12} c_2(M) + \cdots \tag{36}$$

It is first discovered by the requirement of anomaly cancelation on D-brane worldvolume. Combine this charge with the periods obtained from tree level prepotential $F(X^I, \lambda^2 = 0)$ will reproduce the grading formula exactly the same form as what we have derived above in the large volume limit. This strongly suggests this anomaly cancelation factor is from string loop. And our result can be seen as a generalization of this term to the whole moduli space, including all the worldsheet instanton corrections.

2. From the Donaldson-Uhlenbeck-Yau theorem, the sufficient and necessary condition for the existence of the Hermitian-Yang-Mills equation

$$F_{ab} = F_{\bar{a}\bar{b}} = 0, \qquad g^{a\bar{b}} F_{a\bar{b}} = 0 \tag{37}$$

is that the slope of the vector bundle is zero

$$\mu(E) = \frac{1}{rk(E)} \int_M J \wedge J \wedge c_1(E) = 0, \tag{38}$$

which we will call DUY equation. Notice that this is the first term in our formula. Now the α' and string loop corrected equation in the large volume limit is

$$\int_M c_1 \wedge J \wedge J + \frac{\lambda^2}{12} \int_M c_1 \wedge c_2(M) + \frac{1}{3} \int_M (c_1^3 - 3c_1 c_2 + c_3) = 0. \tag{39}$$

The stability condition for a D-brane configuration in type-I theory should be the same, and after a duality map to heterotic string theory can compare the result. The one-loop corrected stability condition for an anomalous line bundle L in heterotic string theory has been found out in [14] by considering gauge and mixed anomaly cancelation, as

$$\int_M c_1(L) \wedge J \wedge J - \frac{1}{2} l_s^2 g_s^2 \int_M (c_1(L) \wedge (\sum_k ch_2(V_k) + \sum_m a_m c_1^2(L_m) + \frac{1}{2} c_2(M)) = 0. \tag{40}$$

Using the anomaly cancelation condition for heterotic string compactification

$$\sum_k ch_2(V_k) + \sum_m a_m c_1^2(L_m) = -c_2(M) \tag{41}$$

this simplifies to

$$\int_M c_1(L) \wedge J \wedge J + \frac{1}{4} l_s^2 g_s^2 \int_M c_1(L) \wedge c_2(M) = 0. \tag{42}$$

After the heterotic-type-I duality

$$\begin{aligned} e^{\phi_{10}^I} &= e^{-\phi_{10}^H}, \\ J^I &= J^H e^{-\phi_{10}^H}, \end{aligned} \tag{43}$$

this maps to an equation in type-I quantity

$$\int_M c_1(L) \wedge J \wedge J + \frac{1}{4} l_s^2 \int_M c_1(L) \wedge c_2(M) = 0, \tag{44}$$

which is exactly of the form of the first two terms in our equation. Obviously, the MMMS equation, being a tree-level result, would only include the first the the third term in our equation. The second term, being a one-loop effect in heterotic string, is mapped to a loop term in type-I. This picture matches perfectly to the fact that all the higher derivative F-terms of the form $F_g W^{2g}$ which are the origin for the correction to the stability conditions are produced at one-loop level in heterotic string theory under the $\mathcal{N} = 2$ type II-heterotic duality [15]. We regard this as a strong evidence for our claim.

Physically, the effect of the curved space in the large volume limit can be interpreted as quantum two-brane charges of a six brane in curved space produced by $c_2(M)$. This phenomena has appeared in many places, and it is satisfactory that our claim in this limit reproduces this result.

Finally, we point that what have obtained is not directly the loop corrected Hermitian-Yang-Mills equation, but corrections to the existence condition for its solution. If a generalized Donaldson-Uhlenbeck-Yau theorem exists, the loop corrected condition is presumed to give the sufficient and necessary condition for a loop corrected HYM equation. It is in this sense that we claim to have a correction to HYM equation. Certainly it would be interesting to explicitly find out the correction terms to the Hermitian-Yang-Mills equation.

3.2 Loop corrected SLAG

At tree-level, the mirror statement about D-brane stability on A-branes concerns the phase of the restriction of the holomorphic three form Ω

$$Re e^{i\theta} \Omega|_L = 0, \tag{45}$$

for certain constant phase θ. Here L is a submanifold of middle dimension which satisfies

$$\omega|_L = 0. \tag{46}$$

It is obvious that after the inclusion of the string loop correction, the SLAG condition should be corrected as

$$\omega|_L = 0, \qquad Re e^{i\theta} \Omega(\lambda)|_L = 0, \tag{47}$$

using the corrected three-form $\Omega(\lambda)$ defined in the previous section.

4 Conclusions and discussions

In this paper, we have argued that the high-derivative F-terms of the form $F_g W^{2g}$, originated from high string loops, in the presence of the constant graviphoton field strength would give a loop corrected central charge for a configuration of D-branes. This implies loop corrected stability condition for BPS D-branes, which for B-type branes at large volume reduces to loop corrected Hermitian-Yang-Mills equation, and for A-type branes reduces to a deformed special Lagrangian submanifold condition. We have also show that graviphoton field strength can be constant even in the presence of effective potential as required by supersymmetry. We have compared the loop corrected stability condition for B-type branes to the various known results and obtained agreement.

There are several questions arise from this result. First, is the string loop corrections to the D-stability condition present here complete? The blackhole solutions and especially the blackhole entropy countings in the presence of high derivative terms have been studied extensively. The entropy countings have been checked to produce the right result in the known examples. Although it is still a puzzle why this particular class of terms should produce an exact result, the fact that it is exact in known entropy counting lends support to the conjecture that the D-brane stability condition as stated in this paper is exact. Since physically these F-terms are produced by integration out the effective 4d BPS particles obtained from wrapping IIA 2-branes around 2-cycles in Calabi-Yau, one may wonder the effects of integrating out branes wrapping higher dimensional cycles. Certainly more understanding in this direction is desirable.

Second, the similar stability problem can be studied in the dual picture of the D-branes wrapping Calabi-Yau and spanning the spacetime. It is known to show up as D-term in the super-Yang-Mills theory on the D-branes. A microscopy string theory calculation of the D-term in the presence of constant graviphoton field strength should be able to verify our claim.

Finally, a probable lesson from the result of this paper is that supersymmetry breakings on the D-brane could be tied up with supergravity background, and could be quite complicated. The implications to supersymmetry breaking and moduli fixing problem in string phenomenology are certainly worth further study.

This work is supported by NSF grant PHY-0244811 and DOE grant DE-FG01-00ER45832.

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